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Some Results on Modified Mean Labeling of Graphs

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Abstract: Let G be a graph with p vertices and q edges. Let $f : V(G) \to \{1, 2, 3, \ldots, p\}$ be an bijective function. For a vertex labeling f, the induced edge labeling $f^*(e = uv)$ is defined by $f^*(e) = \frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)-1}{2}$ if $f(u) + f(v)$ is odd, then f is called a modified *mean* labeling if $\{f^*(e)/e \in E(G)\} = \{1, 2, 3, \ldots, p - 1\}$ and all are distinct integers. In the present work we investigate modified mean labeling of Paths, Caterpillar and Spider.

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1. Introduction

We consider finite undirected graphs without loops and multiple edges. We denote by $V(G)$ and $E(G)$ the set of vertices and the set of edges of a graph G respectively. Let $|V(G)| = p$ and $|E(G)| = q$ be the number of vertices and the number of edges of G. We follow the notation and terminology of [2] and [6]. Different kinds of graphs were studied by T. Nicholas, S. Somasundaram and V. Vilfred [3]. By a *labeling* [1] we mean a one-to-one mapping that carries a set of graph elements into a set of numbers (usually to positive or non-negative integers), called labels. In this paper we deal with labeling with domain the set of all vertices. Mean labeling was introduced by S. Somasundaram and R. Ponraj [4, 5]. In this paper we introduce the mean labeling of graphs and investigate mean labeling of Paths, caterpillar and Spider. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1. *A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{1, 2 \ldots, q\}$ such that when each edge uv is labelled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd then the resulting edges are distinct.*

Definition 1.2. *A walk in a graph is called a Path in which both the vertices and edges are distinct.*

Definition 1.3. *Caterpillar is a tree with all vertices either on a single central path or distance one away form it. The central path may be considered to be the largest path in the caterpillar, so that both end vertices have valency one.*

Definition 1.4. *A Spider $SP(P_{n,2})$ is a caterpillar $S(X_1, X_2, \ldots, X_n)$ where $X_n = 2$ and $X_i = 0$, $i = 1, 2, \ldots, n - 1$.*

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2. Results on Modified Mean Labeling

We introduce the concept of the modified mean labeling of some graphs which is slightly different from [4] and [5].

Definition 2.1. *A connected graph G with vertex set V and edge set E is said to be Modified mean labeling if there exists a bijective function* $f : V(G) \to \{1, 2, \ldots, p\}$ *such that the induced mapping* $f^* : E(G) \to \{1, 2, \ldots, p-1\}$ *defined by*

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)|-1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

And all these edge labelings are distinct.

Theorem 2.2. *Every Path* P_n, $n \geq 2$ *admits on modified mean labeling.*

Proof. Here Let $|V(G)| = n$ and $|E(G)| = n - 1$. Label the vertices of P_n, $n \geq 2$ by $1, 2, \ldots, n$. Now define the edge labeling $f^* : E(G) \to \{1, 2, \ldots, p-1\}$ by $f^*(v_i v_{i+1}) = i$, $i = 1, 2, \ldots, n$. All these edge labels are distinct. Hence the path P_n, $n \geq 2$ admits on modified mean labeling. □

Example 2.3.

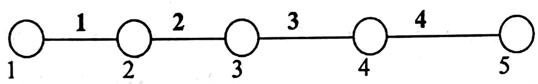

Figure 1. The Path P_5

Theorem 2.4. *The Caterpillar* $T = S(X_1, X_2, \ldots, X_n)$ *has modified mean labeling.*

Proof. The vertex labels are $f(v_i) = 2i - 1$, $i = 1, 2, \ldots, n$ and $f(u_i) = 2i$, $i = 1, 2, \ldots, n$. The induced mapping $f^* : E(G) \to \{1, 2, \ldots, p-1\}$ defined by $f^*(v_i v_{i+1}) = 2i$, $i = 1, 2, \ldots, n$ and $f^*(u_i u_{i+1}) = 2i - 1$, $i = 1, 2, \ldots, n$. All these labels are distinct and hence proved. □

Example 2.5.

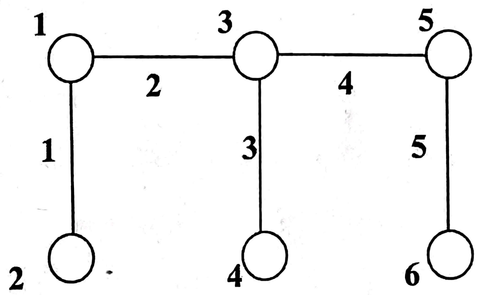

Figure 2. $T = S(X_1, X_2, X_3)$

Theorem 2.6. *The Spider* $SP(P_{n,2})$ *where* $n \geq 2$ *is modified mean labeling.*

Proof. Define f on $V(SP(P_{n,2}))$ by $f(v_i) = i$, $i = 1, 2, \ldots, n$. The induced edge labels are $\{1, 2, \ldots, p-1\}$. Hence proved. □

Example 2.7.

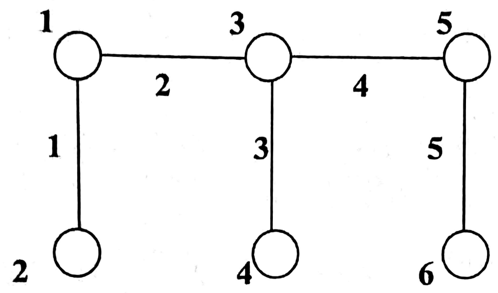

Figure 3. $SP(P_{5,2})$

3. Odd Modified Mean Labeling

We introduce the new concept of odd modified mean labeling and analyse the graphs which are satisfying odd modified mean labeling.

Definition 3.1. *A connected graph G with $|V| = p$ vertices and $|E| = q$ edges is said to be odd-modified mean labeling if there is an bijection $f : V \to \{1, 3, \ldots, 2q + 1\}$ such that the mapping $f^* : E \to \{2, 4, \ldots, 2q\}$ defined by*

$$f^*(v_iv_{i+1}) = \begin{cases} \frac{f(v_i)+f(v_{i+1})}{2} & \textit{if } f(u) + f(v) \textit{ is even} \\ \frac{f(v_i)+f(v_{i+1})-1}{2} & \textit{if } f(u) + f(v) \textit{ is odd} \end{cases}$$

and all these edge labelings are distinct.

Theorem 3.2. *Every Path P_n, $n \geq 2$ admits on odd modified mean labeling.*

Proof. The vertex labelings are $f : V(P_n) \to \{1, 3, 5, \ldots, 2q + 1\}$ and the induced edge labelings are

$$f^*(v_iv_{i+1}) = \begin{cases} \frac{f(v_i)+f(v_{i+1})}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(v_i)+f(v_{i+1})-1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

Hence P_n admits an odd modified mean labeling. □

Theorem 3.3. *The Caterpillar $T = S(X_1, X_2, \ldots, X_n)$ has odd modified mean labeling.*

Proof. The vertices are labeled by odd numbers $\{1, 3, 5, \ldots, 2q + 1\}$. The induced edge labels are $f^* : E(G) \to N$. Thus proved. □

Theorem 3.4. *The Spider $SP(P_{n,2})$ where $n \geq 2$ has odd modified mean labeling.*

Proof. Define f on $V(SP(P_{n,2}))$ by $f(v_i) = i$, $i = 1, 3, 5, \ldots, 2q + 1$. The induced edge labels are $\{1, 2, \ldots, 2p - 3\}$. Hence proved. □

4. Even Modified Mean Labeling

Here we present the new concept of even modified mean labeling and analyse the graphs which are satisfying even modified mean labeling.

Definition 4.1. *A connected graph G with $|V| = p$ vertices and $|E| = q$ edges is said to be even-modified mean labeling if there is an bijection $f : V \rightarrow \{2, 4, \ldots, 2q+2\}$ such that the mapping $f^* : E \rightarrow \{3, 5, \ldots, 2q+1\}$ defined by*

$$f^*(v_iv_{i+1}) = \begin{cases} \frac{f(v_i)+f(v_{i+1})}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(v_i)+f(v_{i+1})-1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

and all are distinct.

Theorem 4.2. *Every Path P_n, $n \geq 2$ admits on even modified mean labeling.*

Proof. Here $f : V(P_n) \rightarrow \{2, 4, 6, \ldots, 2q+2\}$ defined by $f(v_i) = 2i$ and the induced edge labelings are defined by

$$f^*(v_iv_{i+1}) = \begin{cases} \frac{f(v_i)+f(v_{i+1})}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(v_i)+f(v_{i+1})-1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

and also $f^*(v_iv_{i+1}) = 2i+1$. Hence P_n admits an even modified mean labeling. □

Theorem 4.3. *The Caterpillar $T = S(X_1, X_2, \ldots, X_n)$ has even modified mean labeling.*

Proof. The vertices are labeled by even numbers $\{2, 4, 6, \ldots, 2q+2\}$. The induced edge labels are $f^* : E(G) \rightarrow N$. Thus proved. □

Theorem 4.4. *The Spider $SP(P_{n,2})$ where $n \geq 2$ is even modified mean labeling.*

Proof. Define f on $V(SP(P_{n,2}))$ by $f(v_i) = i$, $i = 2, 4, 6, \ldots, 2q+2$. The induced edge labels are $f^* : E(G) \rightarrow N$. Hence proved. □

5. Conclusion

Here we introduced the new concept of modified mean labeling, odd and even modified mean labelings. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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