NEWTON'S FORWARD INTERPOLATION: AN ANOTHER APPROACH

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Abstract:

In order to reduce the numerical computation associated to the repeated application of the existing interpolation formula in computing a large number of interpolated values, a formula has been derived from Newton's forward interpolation formula: an another approach. Application of this formula is given in this paper. This formula is suitable in the situation where the values of the argument are at equal intervals and the value of x starts with origin and the regular intervals should be unit.

Keywords: interpolation, newton's forward interpolation formula, polynomial curve, representation of numerical data

Introduction:

Interpolation, which is the process of computing intermediate values of a function from the set given values of the function. It plays significant role in numerical research almost in all branches of science, humanities, commerce and in technical branches. A number of interpolation formulas namely Newton's forward interpolation formula, Newton's Backward interpolation formula, Lagrange's interpolation formula, Newton's divided difference interpolation formula, Lagrange's interpolation formula, Divided difference interpolation formula, Newton's central Difference interpolation formula, Stirlings formula, Bessel's formula and some others are available in the literature of numerical analysis.

In case of the interpolation by the existing formulae, the value of the dependent variable corresponding to each value of the independent variable in it. That is if it is wanted to interpolate the values of the dependent variable corresponding to a number of values of the independent variable by a suitable existing interpolation formula, it is required to apply the formula for each value separately and thus the numerical computation of the value of the dependent variable based on the given data are to be performed in each of the cases. In order to get rid of these repeated numerical computations from the given numerical data by a suitable corresponding to any given value of the independent variable. However a method/ formula is necessary for representing a given set of numerical data by a suitable polynomial. The formula obtained has been applied to represent a numerical data by a polynomial curve.

Newton's forward interpolation formula for equal intervals:

Theorem

Let the function y=f(x) take the values $y_0, y_1, \ldots\ldots y_n$ at the points $x_0, x_1, \ldots\ldots x_n$ where $x_i = x_0 + ih$. Then, Newton's forward interpolation polynomial is given by

y(x)= $P_n(x) = P_n(x_0 + uh) = y_0 + \frac{u^{(1)}}{1!}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \cdots + \frac{u^{(r)}}{r!}\Delta^r y_0 + \cdots + \frac{u^{(n)}}{n!}\Delta^n y_0$

where, $u^{(r)} = u(u-1)(u-2)\ldots.(u-\overline{r-1})$

(If x is given, u is found out)

$u = \frac{x-x_0}{h}$

Proof:

Suppose the values of x are equidistant, i.e $x_i - x_{i-1} = h$ for i = 1,2,…n

Let $P_n(x)$ be a polynomial of the n^{th} degree in x such that

$$y_i = f(x_i) = P_n(x_i), i = 0,1,2,..,n$$

Assume $P_n(x) = a_0 + a_1(x-x_0)^{(1)} + a_2(x-x_0)^{(2)} + \cdots + a_r(x-x_0)^{(r)} + \cdots + a_n(x-x_0)^{(n)}$ …(1)

The (n+1) unknowns $a_0, a_1, a_2, \ldots a_n$ can be found as follows:

$P_n(x_0) = y_0 = a_0$ setting $x = x_0$ in (1)

$\Delta^r P_n(x) = a_r r!\, h^r + \textit{ terms involving } (x-x_0)$ as a factor …(2)

(since the first r terms vanish)

Set $x = x_0$ in (2)

$\Delta^r P_n(x_0) = a_r r!\, h^r$ (since the other terms in (2) vanish)

i.e $\Delta^r y_0 = a_r r!\, h^r$

$a_r = \frac{1}{r!h^r}\Delta^r y_0$ …..(3)

Using r = 1,2,3…n in (3) ,we get the values of $a_1, a_2, \ldots a_n$

Therefore

$P_n(x) = y_0 + \frac{(x-x_0)^{(1)}}{h}\Delta y_0 + \frac{(x-x_0)^{(2)}}{2!h^2}\Delta^2 y_0 + \cdots \frac{(x-x_0)^{(r)}}{r!h^r}\Delta^r y_0 + \cdots + \frac{(x-x_0)^{(n)}}{n!h^n}\Delta^n y_0$ ..(4)

$$\frac{(x-x_0)^{(r)}}{h^r} = \frac{(x-x_0)(x-x_0-h)(x-x_0-2h)\ldots(x-x_0-\overline{r-1}h)}{h^r}$$

$= \frac{(x-x_0)}{h} \cdot \left(\frac{(x-x_0)}{h} - 1\right)\left(\frac{(x-x_0)}{h} - 2\right) \dots \left(\frac{(x-x_0)}{h} - \overline{r-1}\right)$ where u=$\frac{(x-x_0)}{h}$

$= u^{(r)}$, (here h=1) and $x = x_0 + uh$

Using in (4)

$P_n(x) = P_n(x_0 + uh) = y_0 + \frac{u^{(1)}}{1!}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \cdots + \frac{u^{(r)}}{r!}\Delta^r y_0 + \cdots + \frac{u^{(n)}}{n!}\Delta^n y_0$ ….(5)

Where $u^{(r)} = u(u-1)(u-2) \dots .(u - \overline{r-1})$

(if x is given, u is found out).

Equation (5) is known as Gregory- newton forward interpolation formula.

Alternative approach to find the forward difference values are as follows:

The other way to find the $\Delta^r y_0$ value is as follows

$b_0 = y_0$

$$b_1 = \frac{(y_1 - b_0)}{x_1 - x_0}$$

$$b_2 = \left\{\frac{(y_2 - b_0) - \frac{(x_2 - x_0)}{1!} b_1}{(x_2 - x_0)(x_2 - x_1)}\right\} 2!$$

$$b_3 = \left\{\frac{(y_3 - b_0) - \frac{(x_3 - x_0)}{1!} b_1 - \frac{(x_3 - x_0)(x_3 - x_1)}{2!} b_2}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}\right\} 3!$$

In general we have

$$b_n = \left\{\frac{(y_n - b_0) - \frac{(x_n - x_0)}{1!} b_1 - \frac{(x_n - x_0)(x_n - x_1)}{2!} b_2 \dots \frac{(x_n - x_0)(x_n - x_1)\dots(x_n - x_{n-2})}{n-1!} b_{n-1}}{(x_n - x_0)(x_n - x_1)(x_n - x_2)\dots(x_n - x_{n-1})}\right\} n!$$

Using these values we found the newtons forward difference formula as

$P_n(x) = P_n(x_0 + uh) = b_0 + \frac{u^{(1)}}{1!} b_1 + \frac{u^{(2)}}{2!} b_2 + \cdots + \frac{u^{(r)}}{r!} b_r + \cdots + \frac{u^{(n)}}{n!} b_n$ ….(6)

$u^{(r)} = u(u-1)(u-2) \dots .(u - \overline{r-1})$

NOTE:

This formula is applicable for only equal intervals of x and the value of x starts with origin and unit difference.

Problem:

Using Newton's forward interpolation formula find the cubic polynomial which takes places the following values:

X:	**0**	**1**	**2**	**3**
F(X)	**1**	**2**	**1**	**10**

The forward difference table is

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

The newtons forward formula is

$$y(x) = P_3(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

Here $x_0 = 0, y_0 = 1,\ u = \frac{x-x_0}{h}, h = 1 - 0 = 1, u = x$

$$y(x) = P_3(x) = 1 + \frac{x}{1!}(1) + \frac{x(x-1)}{2!}(-2) + \frac{x(x-1)(x-2)}{3!}(12)$$

$= 1 + x - x(x-1) + 2x(x-1)(x-2)$

$= 1+x - x^2 + x + 2x(x^2 - 3x + 2)$

$= 1+2x - x^2 + 2x^3 - 6x^2 + 4x$

$\boldsymbol{P_3(x) = 2x^3 - 7x^2 + 6x + 1}$

Now using the new method we have to find the values of $b_0, b_1, b_2 \dots b_n$

$b_0 = y_0 \quad , b_0 = 1$

$b_1 = \frac{(y_1 - b_0)}{x_1 - x_0} \quad b_1 = \frac{(2-1)}{1-0},\ b_1 = 1$

$$b_2 = \left\{\frac{(y_2 - b_0) - \frac{(x_2 - x_0)}{1!}b_1}{(x_2 - x_0)(x_2 - x_1)}\right\}2! \ , b_2 = \left\{\frac{(1-1) - \frac{(2-0)}{1!}1}{(2-0)(2-1)}\right\}2!\ ,\ b_2 = \frac{0-2}{2.1}2! \quad , b_2 = -2$$

$$b_3 = \left\{\frac{(y_3 - b_0) - \frac{(x_3 - x_0)}{1!}b_1 - \frac{(x_3 - x_0)(x_3 - x_1)}{2!}b_2}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}\right\}3!$$

$$b_3 = \left(\frac{(10-1) - \frac{(3-0)1}{1!} - \frac{(3-0)(3-1)(-2)}{2!}}{(3-0)(3-1)(3-2)} \right) 3!$$

$$b_3 = \frac{9-3+6}{6}\,6 \;\;,\;\; b_3 = 12$$

So now the we use the new formula we get

$$y(x) = P_3(x) = b_0 + \frac{u}{1!} b_1 + \frac{u(u-1)}{2!} b_2 + \frac{u(u-1)(u-2)}{3!} b_3$$

Here $x_0 = 0, y_0 = 1,\; u = \frac{x-x_0}{h}, h = 1-0 = 1, u = x$

$$y(x) = P_3(x) = 1 + \frac{x}{1!}(1) + \frac{x(x-1)}{2!}(-2) + \frac{x(x-1)(x-2)}{3!}(12)$$

$= 1 + x - x(x-1) + 2x(x-1)(x-2)$

$= 1 + x - x^2 + x + 2x(x^2 - 3x + 2)$

$= 1 + 2x - x^2 + 2x^3 - 6x^2 + 4x$

$\boldsymbol{P_3(x) = 2x^3 - 7x^2 + 6x + 1}$

Hence we got the same solution without finding the newton's forward table.

Conclusion:

The formula described in equation (6) can be use to represent an alternatives to forward difference $\Delta^r y_0$ values by the polynomial curve. The degree of the polynomial is one less than the number of pair of observations. Newton's forward interpolation formula is valid for estimating the value of the dependent variable under the following two conditions:

1. The given values of the independent variable are at equal interval.
2. The value of the independent variable corresponding to which the value of the dependent variable is to be estimated lies in the first half of the series of the givn values of the independent variable.

Therefore, the formula derived here is suitable only for equal intervals of x and the value of x starts with origin and of unit difference.

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