An Extenstion of Fermat's Last Theorem in Seven dimensional Euclidean space

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Abstract

The Fermat last theorem states that there is no integer triple (a,b,c) such that $a^n + b^n = c^n \; for$ n >2. And in an extension of Fermat's last theorem, they shown that $a^n + b^n + c^n = d^n$ is true for n=2,3 and in the extention of an extension of fermat's last theorem proved that $a^n + b^n + c^n + d^n = e^n$ for n=2 and now in this paper it is an attempt to show that $a^n + b^n + c^n + d^n + e^n + f^n + g^n = h^n$ for n=2

Key words: integer sextuples, integer septuple, integer octuple, seven dimension Euclidean space, Fermat last theorem

Introduction

Pierre Fermat (1601-1665) wrote a comment by the side while reading a book of Pythagoras triple that there is no integer triple(a,b,c), for which $a^n + b^n = c^n \; for \; n > 2$ this result known as Fermat's last theorem and unsolved till 1995 when Andrew wiles in a 110-page paper was able to provide a proof [4]

Then in an extension of Fermat's last theorem it is an attempt to extend the Fermat's last theorem to integer quadruple and proved the result for $a^n + b^n + c^n = d^n$ for n=2,3

And in an extension of an extension of fermat's last theorem proved for integer quintuples $a^n + b^n + c^n + d^n = e^n \; for \; n = 2$.

And in an extension of fermat's last theorem in five dimensional Euclidean space prove for integer sextuples $a^n + b^n + c^n + d^n + e^n = f^n \; for \; n = 2$

And in an extension of fermat's last theorem in six dimensional Euclidean space, prove for integer septuples $a^n + b^n + c^n + d^n + e^n + f^n = g^n for \; n = 2$

Now in this paper it is an attempt to solve to solve for integer octuples

$$a^n + b^n + c^n + d^n + e^n + f^n + g^n = h^n for \; n = 2$$

Preliminary results

We present few results on integer octuple

Result 1

If, then multiple of any integer n with this integer octuples is again an integer octuples (na,nb,nc,nd,ne,nf,ng, nh)

Proof

$(na)^2 + (nb)^2 + (nc)^2 + (nd)^2 + (ne)^2 + (nf)^2 + (ng)^2 = n^2a^2 + n^2b^2 + n^2c^2 + n^2d^2 + n^2e^2 + n^2f^2 + n^2g^2$

$=n^2(a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2)$

$=n^2(h^2)$

Result 2

For any octuples (a,b,c,d,e,f,g,h), if a is even and b,c,d,e,f,g are odd then h cannot be an odd

Proof

Let

a =2p, b=2q+1, c=2r+1, d=2s+1 e=2t+1 f= 2l+1,g= 2m+1

for p,q,r,s,t, $l, m \in z$

if $a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 =$

$(2p)^2 + (2q + 1)^2 + (2r + 1)^2 + (2s + 1)^2 + (2t + 1)^2 + (2l + 1)^2 + (2m + 1)^2$

$= 4p^2 + 4q^2 + 1 + 4q + 4r^2 + 1 + 4r + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t + 4l^2 + 1 + 4l + 4m^2 + 1 + 4m$

$= 2(2p^2 + 2q^2 + 2r^2 + 2s^2 + 2t^2 + 2l^2 + 2m^2 + 2q + 2r + 2s + 2t + 2l + 2m) + 6$

Which is an even number

Result 3

For any octuples (a,b,c,d,e,f,g,h), if a,b are even and c,d,e,f ,g are odd then h must be an odd number

Proof :

Let a=2p, b=2q, c=2r+1, d=2s+1, e=2t+1, f=2l+1, g= 2m+1

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2$
$= 4p^2 + 4q^2 + (2r + 1)^2 + (2s + 1)^2 + (2t + 1)^2 + (2l + 1)^2 + (2m + 1)^2$

$= 4p^2 + 4q^2 + 4r^2 + 1 + 4r + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t + 4l^2 + 1 + 4l + 4m^2 + 1 + 4m$

$h^2 \quad = 2(2p^2 + 2q^2 + 2r^2 + 2r^2 + 2t^2 + 2s^2 + 2l^2 + 4m^2 + 2r + 2s + 2t + 2l + 2m) + 5$

Which is an odd number

Result 4

For any octuples (a,b,c,d,e,f,g,h),if a,b,c are even and d, e, f,g are odd then h must be an even

Proof

Let a=2p, b=2q, c=2r, d=2s+1, e= 2t+1, f=2l+1, g= 2m+1

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2$
$= (2p)^2 + (2q)^2 + (2r)^2 + (2s + 1)^2 + (2t + 1)^2 + (2l + 1)^2 + (2m + 1)^2$

$= 4p^2 + 4q^2 + 4r^2 + 4s^2 + 1 + 4s + 4t^2 + 1 + 4t + 4l^2 + 1 + 4l + 4m^2 + 1 + 4m$

$h^2 \quad =4p^2 + 4q^2 + 4r^2 + 4s^2 + 4t^2 + 4l^2 + 4m^2 + 4s + 4t + 4l + 4m + 4$

An even number

Result: 5

For any octuples (a, b, c, d, e, f ,g,h),if a, b, c, d are even and e,f,g are odd then h must be odd

Proof

Let a=2p, b=2q, c=2r, d=2s, e= 2t+1, f=2l+1, g= 2m+1

$$a^2+b^2+c^2+d^2+e^2+f^2+g^2=4p^2+4q^2+4r^2+4s^2+4t^2+4t+1+4l^2+1+4l+4m^2+1+4m$$

= $2(2p^2+2q^2+2r^2+2s^2+2t^2+2l^2+2m^2+2t+2l+2m+1)$+1

$h^2\ = an\ odd\ number$

Result 6

For any octuples (a, b, c, d, e, f,g,h), if a, b, c, d, e are even and f,g is odd then h must be an even

Proof

Let a=2p b=2q c=2r, d=2s, e= 2t, f=2l+1, g= 2m+1

$$a^2+b^2+c^2+d^2+e^2+f^2+g^2=4p^2+4q^2+4r^2+4s^2+4t^2+4l^2+1+4l+4m^2+1+4m$$

=$2(2p^2+2q^2+2r^2+2s^2+2t^2+2l^2+2m^2+2l+2m+1)$

$h^2\ = an\ even\ number$

Result 7

For any octuples (a, b, c, d, e, f, g, h), if a, b, c, d, e, f are even and g is odd then h must be odd

Proof

Let a=2p b=2q c=2r, d=2s, e= 2t, f=2l, g= 2m+1

$$a^2+b^2+c^2+d^2+e^2+f^2+g^2=4p^2+4q^2+4r^2+4s^2+4t^2+4l^2+4m^2+1+4m$$

= $2(2p^2+2q^2+2r^2+2s^2+2t^2+2l^2+2m^2+2m)$+1

an odd number

Result 8

For any octuples (a, b, c, d, e, f, g, h), if a, b, c, d, e ,f, g are odd then h must be odd

Proof:

a =2p+1, b=2q+1, c=2r+1, d=2s+1 e=2t+1, f=2l+1, g= 2m+1

for p, q, r, s, t, $l, m \in z$

if $a^2+b^2+c^2+d^2+e^2+f^2+g^2=$

$$(2p+1)^2+(2q+1)^2+(2r+1)^2+(2s+1)^2+(2t+1)^2+(2l+1)^2+(2m+1)^2$$

$$=4p^2+4p+1+4q^2+1+4q+4r^2+1+4r+4s^2+1+4s+4t^2+1+4t+4l^2+1+4l+4m^2+1+4m$$

$$=2(2p^2+2q^2+2r^2+2s^2+2t^2+2l^2+2m^2+2p+2q+2r+2s+2t+2l+2m+3)+1$$

Which is an odd number

Result 9

For any octuples (a, b, c, d, e, f, g, h), if a, b, c, d, e, f ,g are even then h must be even

Proof

Let a=2p b=2q c=2r, d=2s, e= 2t, f=2l, g= 2m

$$a^2+b^2+c^2+d^2+e^2+f^2+g^2=4p^2+4q^2+4r^2+4s^2+4t^2+4l^2+4m^2$$

$$=2(2p^2+2q^2+2r^2+2s^2+2t^2+2l^2+2m^2+)$$

An even number

Main result

Integer octuples and the seven dimensional Euclidean

Here we relate the integer octuples (a,b,c,d,e,f,g,h) to points on seven dimensional Euclidean space and a solution is obtain for the equation $x^2+y^2+z^2+u^2+v^2+w^2+o^2=1$ from we get the general solution for integer octtuples is $a^2+b^2+c^2+d^2+e^2+f^2+g^2=h^2$

Let (a, b, c, d, e, f, g, h), be integer octuples for which $a^2+b^2+c^2+d^2+e^2+f^2+g^2=h^2$

Divide by h^2, we get $\left(\frac{a}{h}\right)^2+\left(\frac{b}{h}\right)^2+\left(\frac{c}{h}\right)^2+\left(\frac{d}{h}\right)^2+\left(\frac{e}{h}\right)^2+\left(\frac{f}{h}\right)^2+\left(\frac{g}{h}\right)^2=1$

$\left(\frac{a}{h},\frac{b}{h},\frac{c}{h},\frac{d}{h},\frac{e}{h},\frac{f}{h},\frac{g}{h}\right)$ *is the* Solution of the equation $x^2+y^2+z^2+u^2+v^2+w^2+o^2=1$

Here $x^2+y^2+z^2+u^2+v^2+w^2+o^2=1$ is an 7 dimensional Euclidean space whose coordinates (x, y, z, u, v, w,o) are rational number. Notice that it has 14 coordinates' points $(\pm1,0,0,0,0,0,0), (0,\pm1,0,0,0,0,0), (0,0,\pm1,0,0,0,0), (0,0,0,\pm1,0,0,0), (0,0,0,0,\pm1,0,0)$ *and*

$(0,0,0,0,0,\pm1,0)$, $(0,0,0,0,0,0,,\pm1)$

Suppose we consider a vector b and the line L going through the point $((-1,0,0,0,0,0)$ having b as its direction. The line L is given by the vector equation

$$L: r=-i+tb$$

Where b=$b_1 i+b_2 j+b_3 k+b_4 l+b_5 m+b_6 n+b_7 \alpha$ the Cartesian equation of L is given by

$\frac{x+1}{b_1}=\frac{y}{b_2}=\frac{z}{b_3}=\frac{u}{b_4}=\frac{v}{b_5}=\frac{w}{b_6}=\frac{o}{b_7}$ To find the intersection of space and L, we have to solve

$x^2+y^2+z^2+u^2+v^2+w^2+o^2=1$ and $\frac{x+1}{b_1}=\frac{y}{b_2}=\frac{z}{b_3}=\frac{u}{b_4}=\frac{v}{b_5}=\frac{w}{b_6}=\frac{o}{b_7}$

From the above equation we get

Y=$\frac{b_2(x+1)}{b_1}$, z=$\frac{b_3(x+1)}{b_1}, u=\frac{b_4(x+1)}{b_1}, v=\frac{b_5(x+1)}{b_1}, w=\frac{b_6(x+1)}{b_1}, o=\frac{b_7(x+1)}{b_1}$

Now substitute these values in Euclidean space equation we have

$$x^2+\left(\frac{b_2(x+1)}{b_1}\right)^2+\left(\frac{b_3(x+1)}{b_1}\right)^2+\left(\frac{b_4(x+1)}{b_1}\right)^2+\left(\frac{b_5(x+1)}{b_1}\right)^2+\left(\frac{b_6(x+1)}{b_1}\right)^2+\left(\frac{b_7(x+1)}{b_1}\right)^2=1$$

$${b_1}^2x^2+{b_2}^2(x+1)^2+{b_3}^2(x+1)^2+{b_4}^2(x+1)^2+{b_5}^2(x+1)^2+{b_6}^2(x+1)^2+{b_7}^2(x+1)^2={b_1}^2$$

$$\left({b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2++{b_7}^2\right)x^2+2\left({b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2\right)x+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2={b_1}^2$$

$$\left({b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2\right)(x^2+2x+1)=2{b_1}^2(x+1)$$

$$\left({b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2\right)(x+1)^2=2{b_1}^2(x+1)$$

$$\left({b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2\right)(x+1)=2{b_1}^2$$

$$x=\frac{{b_1}^2-{b_2}^2-{b_3}^2-{b_4}^2-{b_5}^2-{b_6}^2-{b_7}^2}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2}$$

Similarly the value of y , z, u, v, w and o are

$$y=\frac{2b_1b_2}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},$$

$$z=\frac{2b_1b_3}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},$$

$$u=\frac{2b_1b_4}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},$$

$$v=\frac{2b_1b_5}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},$$

$$w=\frac{2b_1b_6}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2}$$

And

$$o=\frac{2b_1b_7}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},$$

Thus every point (x, y, z, u, v , w, o) on the 7 dimensional Euclidean space

$$x^2+y^2+z^2+u^2+v^2+w^2+o^2=1$$

Is $\left(\frac{{b_1}^2-{b_2}^2-{b_3}^2-{b_4}^2-{b_5}^2-{b_6}^2-{b_7}^2}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},\frac{2b_1b_2}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},\right.$
$\frac{2b_1b_3}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},\frac{2b_1b_4}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},$
$\left.\frac{2b_1b_5}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},\frac{2b_1b_6}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2},\frac{2b_1b_7}{{b_1}^2+{b_2}^2+{b_3}^2+{b_4}^2+{b_5}^2+{b_6}^2+{b_7}^2}\right)$

Now we give a result which obtain the octuples (a, b, c, d, e, f, g, h)

Theorem

For any integer $a=p^2-q^2-r^2-s^2-t^2-l^2-m^2,\ b=2pq, c=2pr, d=2ps, e=2pt, f=2pl, g=2pm\ for\ p,q,r,s,t,l,m\in z$ then

$h=p^2+q^2+r^2+s^2+t^2+l^2+m^2,$ Will satisfies the equation

$$a^2+b^2+c^2+d^2+e^2+f^2+g^2=h^2$$

Proof

Now $a^2+b^2+c^2+d^2+e^2+f^2+g^2=(p^2-q^2-r^2-s^2-t^2-l^2-m^2)^2+(2pq)^2+(2pr)^2+(2ps)^2+(2pt)^2+(2pl)^2+(2pm)^2$

$=\left(p^2-(q^2+r^2+s^2+t^2+l^2+m^2)\right)^2+(2pq)^2+(2pr)^2+(2ps)^2+(2pt)^2++(2pl)^2+(2pm)^2$

$=p^4 - 2(q^2 + r^2 + s^2 + t^2 + l^2 + m^2)p^2 + (q^2 + r^2 + s^2 + t^2 + l^2 + m^2)^2 + 4p^2q^2 + 4p^2r^2 + 4p^2s^2 + 4p^2t^2 + 4p^2l^2 + 4p^2m^2$

$$=p^4 + 2(q^2 + r^2 + s^2 + t^2 + l^2 + m^2)p^2 + (q^2 + r^2 + s^2 + t^2 + l^2 + m^2)^2$$

$$=(p^2 + q^2 + r^2 + s^2 + t^2 + l^2 + m^2)^2 = h^2$$

Conclusion

In this paper I extend the Fermat's last theorem and an attempt made to produce the result for $a^n + b^n + c^n + d^n + e^n + f^n + g^n = h^n$ for n=2, thus we called an extension of Fermat's last theorem in seven dimensional Euclidean space

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